

Injection Locking and Modulation

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Injection Locking

$$V = Ae^{j\theta} = Ae^{j\int \omega(t)dt}$$

Adler’s Equation:

$$\frac{d\theta}{dt} = \omega_o + \left| \frac{\omega_o}{2Q} \frac{A_{inj}}{A} \right| \sin(\theta_{inj} - \theta)$$

$$\frac{d\theta}{dt} \approx \omega_o + \Delta\omega_{lock}(\theta_{inj} - \theta)$$

where $\omega_o = \omega_o t + \phi$

Injection Locking (cont.)

$$\frac{d\theta}{dt} + \Delta\omega_{\text{lock}} = \omega_o + \Delta\omega_{\text{lock}} \int \omega_{\text{inj}}(t) dt$$

$$\frac{d^2\theta}{dt^2} + \Delta\omega_{\text{lock}} \frac{d\theta}{dt} = \Delta\omega_{\text{lock}} \omega_{\text{inj}}(t)$$

$$\frac{d\omega}{dt} + \Delta\omega_{\text{ck}} = \Delta\omega_{\text{lock}} \omega_{\text{inj}}$$

Linear Chirp Modulation

If

$$\omega_{\text{inj}}(t) = \omega_{\text{m}} + at$$

then

$$\omega(t) = \omega_{\text{inj}} - \frac{a}{\Delta\omega_{\text{lock}}}$$

Linear Chirp (cont.)

- Constant frequency offset leads to linear phase offset.
 - This eventually results in loss of lock.
 - Before loss of lock, the assumption of small sine function argument is violated.
- Nevertheless, the calculation is informative:

Sinusoidal Frequency Modulation

If

$$\omega_{\text{inj}}(t) = \omega_c + \alpha \cos(\omega_m t)$$

then

$$\omega(t) = \omega_c + \alpha \cos[\omega_m (t - t_d)]$$

where

$$t_d = \frac{1}{\Delta\omega_{\text{lock}}}$$

Sinusoidal FM (cont.)

- Lock can be maintained if modulation index is small.
- Small sine argument assumption is valid for small modulation index.
 - Integral of frequency offset is sinusoidal.
 - Phase deviation is limited.